Sediment Transport Calculation Using the Ackers-White Method in River with Compound Cross-Section

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ABSTRACT: The presented research concerns analysis of the dependence of the threshold parameter value on the cross-section shape of the river and particularly in the case of the compound cross-section. The Ackers-White method was applied for the sediment stream calculation. A very sensitive parameter of this method is the one representing threshold of sediment movement. Based on the analysis, it was possible to obtain the changes of the threshold parameter value versus depth and flow. Another problem is that the Ackers-White method originally was developed for a uniform flow in the river. In the case of the Lower Oder River there is the Baltic Sea water level influence and the backwater curve. As an example, practical calculations concerning case study: the Oder River close to Gryfino town are given. Close to the river there is a coal driven Electro Power Plant “Dolna Odra”. The ashes are accumulated on the terrace of the compound river cross-section. Research is made to check what the mechanism of mixing natural sediment stream and the ashes from waste stacks is in case of flood.

1 INTRODUCTION

The paper presents an analysis of sediment transport at the outflow distance of the Odra River. The Oder River mouth is a very complicated hydrographical net. Going downstream, the river splits in two branches at Widuchowa town: the Western Oder (the original riverbed) and the Eastern Oder which is a man-made canal facilitated to water transport. Before the river reaches the sea there is a large water reservoir Szczecin Lagoon. Szczecin Lagoon is connected with Baltic Sea with three passes. The river net is shown on the Fig. 1

The area along the river is densely populated and industrialized. Part of the area is devoted to the natural preservation park (the Lower Oder River Park). At the Department of Geotechnology of the West Pomeranian University of Technology, the research has been carried on concerning flow and sediment transport modeling in the Lower Oder River. The average annual flow is equal to 450 m$^3$/s and the free water surface slope is very small, even $10^{-5}$. The river flow is strongly influenced by the sea level. Due to the sea water level changes, backwater curve in river mouth is created.

The detailed pieces of informations about flow characteristics in the Lower Oder River are given in the monography by Coufal (1997). Several papers have been elaborated on flow phenomenon at this river mouth: salt water wedge penetration upstream the river by Meyer & Pluta (2001); wind influence on the backwater curve in the river outlet by Meyer & Coufal (1993), and dependence of Maning roughness coefficient on the bottom sediment composition Kotiasz (2000). The summary of sediment transport research in the Lower Oder River was made by Pluta (2003). It comes out from this evaluation that the Ackers-White(1975) method gives the best fit at the outlet distance of the Oder River. Sediment stream calculations using Ackers-White method are presented in the following papers: Meyer & Skorupska (2005), Meyer & Coufal (2007) and Meyer & Krupański (2008), all concerning the Oder River.
The above research included also estimation of representative sediment diameter, which should be taken to the calculations. The following basic conclusions can be drawn from the aforementioned research:
- the best results of sediment stream calculation were obtained using the Ackers-White method,
- the most sensitive parameter in the Ackers-White model is the one defining sediment flow threshold,
- the free water surface slope is very small and difficult to be measured in the considered case of the Lower Oder River, so an independent variable was taken in the mathematical description of flow, and
- the Ackers-White method concerns the case of the river cross-section with concentrated shape, it does not cover the case of compound cross-section.

The Lower Oder River has a compound cross-section. It means that there are a main flow and a flood area bounded with dikes. At Gryfino cross-section there is Electro Power Plant Dolna Odra. It is the coal powered plant, producing ashes. The ashes wastes are accumulated in stacks which are placed on the flood area.

The question appears how the flowing water washes out the ashes from the stacks during the flood time. A more general problem arises how to describe the presence of ashes in the sediment stream using the Ackers-White method.

2. MATHEMATICAL DESCRIPTION

The calculations of sediment transport stream were based upon the Ackers-White formulae. The advantage of this method is the fact that:
- it includes water flow factors, i.e. the shear velocity to the mean velocity ratio. It allows to include the backwater curve, which exists at the river outlet due to the sea water level, in the calculations,
- the sediment stream calculation refers to the total sediment, i.e. bed and suspended load, and
- the numerical procedure allows to include all fractions of sediment in the case when the sediment composition i.e. sieve curve is based upon the samples taken from the river bottom.

The following symbols were introduced into the above relationships:
\[ D = \text{sediment diameter}; \quad A,C,n,m = \text{constant values depending on } D_g; \quad D_g = \text{dimensionless grain diameter}; \quad H = \text{water depth}; \quad S = \text{sediment grain density to water density ratio}; \quad \alpha = \text{constant value (according to Ackers } \alpha = 12.3); \quad F_g = \text{mobility function}; \quad v_0 = \text{mean water velocity}; \quad \nu_s = \text{shear velocity}. \]

The relationship for constant parameters takes from:
\[ n=\begin{cases} 1.0 & \text{for } D_{gr} \leq 1 \\ 1-0.56 \log(D_{gr}) & \text{for } D_{gr} \leq 60 \\ 0 & \text{for } D_{gr} > 60 \end{cases} \quad (5) \]

\[ m=\begin{cases} 9.66/D_{gr} + 1.34 & \text{for } D_{gr} \leq 60 \\ 0.17 & \text{for } D_{gr} > 60 \end{cases} \quad (6) \]

The Ackers-White method is described by the following relationship:
\[ \omega = \rho \cdot g \cdot Q \cdot X \quad (1) \]

where \( g = \text{acceleration due to the gravity}; \quad Q = \text{flow in the river}; \quad X = \text{dimensionless factor of sediment transport}; \quad \rho = \text{water density}; \quad \omega = \text{sediment stream}. \]

The dimensionless factor of sediment transport results from:
\[ X = \frac{S \cdot D}{H} \left( \frac{v_0}{\nu_s} \right)^n \cdot G_{gr} \quad \text{where} \quad (2) \]

\[ G_{gr} = C \left( \frac{F_g}{A} - 1 \right)^m; \quad D_{gr} = D \cdot \left[ \frac{g(S-1)}{v^2} \right]^{1/3} \quad (3) \]

\[ F_g = \frac{v_0}{\sqrt{g \cdot D \cdot (S-1) \cdot \frac{\alpha H}{D}}} \cdot \left[ \frac{v_0}{\nu_s} \cdot 32 \cdot \log \frac{\alpha H}{D} \right]^n \quad (4) \]
The above parameter $A$ in Eq. 7, represents the threshold of sediment movement. Ackers suggests to take one representative grains diameter for the whole sediment sample for practical calculations, i.e. $D = D_{35}$.

If the calculation is proceeded for all fractions the sieve curve must be included $\{D_i, p(D_i)\}$ and the total sediment stream results from the bellows equation:

$$\omega = \sum [p(D_i) \cdot \omega(D_i)]$$

According to the field measurements Ackers limits this method to the mineral sediment (no cohesive) with grains diameter $D < 0.05\text{mm}$.

3. FLOW CONDITIONS ASSUMED FOR CALCULATIONS

The compound river cross-station was assumed for further calculations, i.e. the main stream and the flood terrace. In order to simplify the procedure a rectangular shape was adopted Fig. 2

![Figure 2. Compound river cross-section](image)

It is also assumed in the further calculations that each width of three parts of the cross-section is known: $B_1, B_2, B_3$.

The free water surface level is also known, so the depth in each part is: $H_1, H_2, H_3$.

The Maning roughness coefficients $n_i$ vary in each part and they are given as: $n_{i1}, n_{i2}, n_{i3}$. These coefficients at the flood terrace should be calculated according to the existing surface, including soil, grass, and bushes. The $n_{i2}$ value for the Lower Oder River can be calculated according to the earlier research, Kotiasz (2000) from the following empirical equation

$$n_{i2} = 0.111 \cdot D^{1/6}$$

where $D$ is bottom grain diameter in $[\text{m}]$.

In the case when sieve curve of bottom sediment material is known, the Maning roughness coefficient $n_{i2}$ is equal to:

$$n_{i2} = 0.111 \cdot \sum [p(D_i) \cdot D_{i/6}]$$

The relation between discharge and flow conditions was assumed according to the Chezy formula:

$$Q = Q_1 + Q_2 + Q_3 = \sum_{j=1}^{3} Q_j \quad \text{where}$$

$$Q_j = \frac{B_j}{n_{ij}} \cdot H_j^{5/3} \cdot \sqrt{I} \quad \text{and} \quad j = 1, 2, 3$$

In the above equations $I$ denotes the free water surface slope in the river. In case of the Lower Oder River the slope is very small even $10^{-5}$, so it is difficult to measure it directly. Because of that, the total flow $Q$ was assumed as the independent variable in further analysis. So, the following relation can be drawn:

$$\sqrt{I} = \frac{Q}{\sum_{j=1}^{3} \left( \frac{B_j}{n_{ij}} \cdot H_j^{5/3} \right)}$$

and the partial flows $Q_j$ are equal to:

$$Q_j = Q \cdot \frac{\frac{B_j}{n_{ij}} \cdot H_j^{5/3}}{\sum_{j=1}^{3} \left( \frac{B_j}{n_{ij}} \cdot H_j^{5/3} \right)}$$

On this way it is possible to omit calculating the value $I$ determined by the backwater curve after assuming the independent variable value $Q$, the slope is included indirectly by Eq. (14).
4. CALCULATION PROCEDURE

The calculations of total sediment stream are based on the Ackers-White method. The sediment stream was calculated for each part of the cross section. So we have

$$\omega = \omega_1 + \omega_2 + \omega_3 = \sum_{j=1}^{3} \omega_j \quad (16)$$

In practical cases the resulting value \( \omega \) is expressed in \([g/s]\). The partial sediment streams are calculated according to the sieve curve of bottom material. So, it gives

$$\omega_j = \sum_{i=1}^{3} \left[ p_i(D_j) \cdot \omega_j(D_j) \right] \quad (17)$$

It the above equation the relationships (2)÷(8) and 12 ÷15 can be applied. Hence it is

$$\omega_j(D,J) = \rho \cdot g \cdot Q_j \cdot X_j(D,J) \quad (18)$$

Furthermore we have

$$v_{ij} = \frac{Q_j(Q)}{B_j \cdot H_j} \quad \text{and} \quad v_{ij} = \sqrt{gH_j \cdot J(Q)} \quad (19)$$

and

$$A,C,n,m = f[D_{ij}(D_j)] \quad (20)$$

it has been assumed for practical calculations

$$\alpha = 12.3 \quad \text{and} \quad \nu = 1.14 \cdot 10^{-6} \text{ m}^2 / \text{s}$$

The above procedure allows to calculate the partial and the total sediment streams using, sediment composition, cross-section shape, depth and flow. The procedure assumes that the sediment stream in each part is independent from each other.

In practice there is a sediment exchange between the main stream and the terraces. In order to include the sediment exchange among three different parts of the river cross-section in this paper it has been suggested to assume A value as varying. This value indicates the threshold of the sediment movement in the Ackers-White formulae. One representative sediment composition exists for the whole cross-section in the case of compound river cross-section. The representative sediment sieve curve \( \{p_0(D_j),D_j\} \) is defined as follows:

$$p_0(D_j) \cdot \omega_0(D_j) = \sum_{j=1}^{3} \left[ p_j(D_j) \cdot \omega_j(D_j) \right] \quad (21)$$

$$\omega_0(D_j) = \sum_{j=1}^{3} \omega_j(D_j) \quad \text{and so} \quad (22)$$

In fact the above principle says that regarding to the \( D_j \) fraction the sum of sediment stream in three parts of the cross-section is the same as in the whole stream.

The abovementioned parameter \( A \) appears in the equations (21); (22) and (23). In order to express the fact that now, it varies, the following formula was introduced

$$A_i = A \cdot \varepsilon \quad (24)$$

The parameter \( A \) is the original one from the Ackers-White method (Eq.7) and \( \varepsilon \) is a parameters, to be calculated and it varies practically from 0.3 to 1.4 depending on flow conditions. The parameter \( \varepsilon \) results from the additional following principle:

$$\sum_{j=1}^{3} \left[ p_0(D_j) \cdot \omega_0(D_j) \cdot 1 \quad (25)$$

The sum covers all fractions \( D_i \) in the above equation.

5. PRACTICAL CALCULATIONS

In order to facilitate calculations using of the above method, the river cross section Gryfino for the Oder River was taken.

The geometry of the river is given in Table 1. In order to simplify the calculation, the rectangular shape for each part was assumed (see Fig. 2).

Table 1. The dimensions of the river cross-section

<table>
<thead>
<tr>
<th>Case No</th>
<th>( B_1[m] )</th>
<th>( B_2[m] )</th>
<th>( B_3[m] )</th>
<th>( H_1[m] )</th>
<th>( H_2[m] )</th>
<th>( H_3[m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>80</td>
<td>120</td>
<td>1.0</td>
<td>3.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>80</td>
<td>120</td>
<td>2.0</td>
<td>4.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>80</td>
<td>120</td>
<td>1.0</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>80</td>
<td>40</td>
<td>1.0</td>
<td>3.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>40</td>
<td>2.0</td>
<td>1.0</td>
<td>4.0</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 2. The composition of the bottom sediment

<table>
<thead>
<tr>
<th>( D_j[m] )</th>
<th>( j = 1 )</th>
<th>( j = 2 )</th>
<th>( j = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00005</td>
<td>0.21</td>
<td>0.10</td>
<td>0.19</td>
</tr>
<tr>
<td>0.00010</td>
<td>0.35</td>
<td>0.19</td>
<td>0.32</td>
</tr>
<tr>
<td>0.00050</td>
<td>0.29</td>
<td>0.38</td>
<td>0.28</td>
</tr>
<tr>
<td>0.00100</td>
<td>0.10</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td>0.00250</td>
<td>0.05</td>
<td>0.12</td>
<td>0.06</td>
</tr>
</tbody>
</table>
The calculation results \( \varepsilon = \varepsilon(Q) \) and appropriate \( \omega_0 = \omega_0(Q) \) are given in Fig. 3

\[ Q = \omega_0 \varepsilon \]

In Fig. 3 the dot line represents the case when \( H_z = 4m \) and the solid line is \( H_z = 3m \) when \( B_z = 120m \)

The calculations were proceeded also for the case if on the eastern terrace \( (j = 3) \), the sediment consists only of ashes. The conditions assumed for this calculations are given in Table 3 and the assumed sediment composition is given in Table 4.

<table>
<thead>
<tr>
<th>( j )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_j [m] )</td>
<td>120</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>( H_j [m] )</td>
<td>1.0</td>
<td>4.0</td>
<td>2.0</td>
</tr>
<tr>
<td>( n_{sj} )</td>
<td>0.057</td>
<td>0.033</td>
<td>0.080</td>
</tr>
</tbody>
</table>

Table 4. The sediment composition in the case of presence of ashes

| \( D_j [m] \) | \( p_j(D_j) \) |
|---|---|---|
| \( j = 1 \) | \( j = 2 \) | \( j = 3 \) |
| 0.00005 | 0.20 | 0.10 | 0.85 |
| 0.00010 | 0.40 | 0.20 | 0.15 |
| 0.00050 | 0.30 | 0.50 | 0 |
| 0.00100 | 0.10 | 0.15 | 0 |
| 0.00200 | 0 | 0.05 | 0 |

In this case the influence of the sea water level and the existing backwater curve are included. The additional parameter \( \xi \) was introduced as the ratio of the free water surface slope in case of backwater \( I_1 \), to the slope \( I \) for uniform flow, then it is

\[ \xi = \frac{I_1}{I} \]  

From analysis of Bernoulli equation the following relation can be derived for the slope under the backwater curve conditions:

\[ \frac{I_{j-1}}{I} = 1 - (1 - \xi) \left( \frac{1 - \beta \cdot Q_j^3}{g \cdot B_j^5 \cdot H_j^9} \right) \]  

and therefore in the Ackers \( F_{gr} \) formula Eq. 4 it is

\[ \frac{\nu_j}{\nu_{oj}} = n_{sj} \cdot \sqrt{\frac{g}{H_j^{1/6}}} \cdot \sqrt{\frac{I_{sj}}{I}} = f(Q, H, \xi) \]  

In the above equations: \( \beta = 1.1 \) is the Saint Venant and \( I_1 \) = shear slope, which is equivalent to the shear velocity, Eq.19.

The above formulae should be applied to the each part of the compound river cross-section separately.

The calculation results for the case when sediment contains also ashes is given in Fig. 4.
Fig 4. The results of calculations at $\xi(Q)$, $\omega_0(Q)$, and $\omega_3(Q)$ for the case when sediment contains ashes

The case when the sediment contains ashes was chosen, to check not only the carrying capacity of the sediment stream at the eastern terrace, but also the possibility to borrow the ashes from the stack by flowing water in the case of flood. It turns out that even 500 tones of ashes can be taken away daily.

6. CONCLUSIONS

6.1 The paper presents a method of sediment stream calculation using the Ackers-White method in case when there is a compound river cross section. The method includes optimization of parameter $A$ which represents the threshold of sediment movement.

6.2 The method allows to calculated the sediment stream for the given sediment composition at each part of the cross-section. It includes also the influence of the sea level and the resulting backwater curve which changes the free water surface slope. In this way it was possible to estimate the absorption of ashes by flowing water during the flood time.

6.3 In order to formulate more precise conclusions concerning resulting sediment stream and partial streams the program of further research foresees field measurements at the Gryfino cross-section.

REFERENCES


