METHOD OF CALCULATING THE SUBSIDENCE OF ORGANIC SOIL BURDENED WITH ASH-SLAG MIXTURE

1. Introduction

In the theory of soil consolidation it is assumed, that a consolidated layer is subjected to some external loading, which initiates the phenomenon of forced filtration. The external load may be effected as an overburdening embankment made of mineral material, or for example ash-slag mixture. In the case of a permeable mineral material overburdening a layer of weak soil, excess of water will be drained upwards to the surface, and this constitutes vertical filtration. When the overburdening material is not permeable, water will be flowing out towards areas, where water pressure is considerably lower, that is outside the footprint of the overburdening embankment, and this constitutes horizontal filtration. Groundwater escaping from the pores of soil gives room for reducing the porosity, and this reduced volume is leading to the subsidence of ground. Terzaghi’s theory of consolidation describing this phenomenon, assumes, that the consolidated soil is not changing its parameters in time, and groundwater is filtrating vertically. Analytical and field investigations, which have been carried out in the Chair of Geotechnics of Szczecin Polytechnic [1, 2] are indicating, that with changes in the soil porosity caused by subsidence, shear (compressibility) modulus is rising, and the hydraulic conductivity in peat is falling down. The increase in shear modulus along with consolidation is resulting in final subsidence of peat layer being smaller, than would be expected according to the linear theory, whereas reduction of the hydraulic conductivity is slowing down the consolidation process. Change in the hydraulic conductivity does not affect the dimension of subsidence, but is expanding the time after which a given level of subsidence is reached, in comparison with a constant hydraulic conductivity. The purpose of this
paper is to demonstrate possibilities of employing Terzaghi’s theory in
calculations of subsidence, when two-directional horizontal outflow of
filtration water – outside the footprint of an overburdening embankment –
is considered.

2. Calculational model of soil

Terzaghi’s equation is a basic equation describing the process of
consolidation [3]:

\[
s(t) = s_\infty \left\{ 1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \left[ \frac{1}{(2n+1)^2} \cdot \exp \left\{ -\left(\frac{2n+1}{4} \cdot \frac{\pi^2}{T_0} \right)^2 \right\} \right]\right\}
\]

\[ (1) \]

![Consolidation model according to Terzaghi](image)

In the case of one-directional vertical filtration, which may be
assumed if the dimensions of the overburdening structure are much
greater than the thickness of the consolidated layer of weak soil, and if
overburdening layer is permeable, then time constant \( T_0 \) depends on the
thickness of the consolidated layer \( H \). When water is flowing out
vertically upwards from the consolidated layer, than:

\[
T_0 = \frac{\gamma_w \cdot H^2}{M \cdot k}
\]

\[ (2) \]
Respectively for a horizontal flow, water leaving the consolidated layer horizontally must travel the distance of width and length of the overburdened area.

Because of this the time constant will then be (3):

$$T_{0L} = \frac{\gamma_w \cdot \left(\frac{L}{2}\right)^2}{M_0 \cdot k}$$

And respectively:

$$T_{0B} = \frac{\gamma_w \cdot \left(\frac{B}{2}\right)^2}{M_0 \cdot k}$$

Assuming in Terzaghi’s solution, that the run of the subsidence may be expressed in a following manner:

$$s = s_\infty \cdot \left\{ 1 - F\left(\frac{t}{T_0}\right) \right\} = s_\infty \cdot \left\{ 1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \left[ \frac{1}{(2n+1)^2} \cdot \exp\left(-\left(2n+1 \right)^2 \cdot \frac{\pi^2}{4} \cdot \frac{t}{T_0}\right) \right] \right\}$$

then in describing consolidation in vertical direction, we will apply the time constant bound with the thickness of the consolidated layer, while in describing consolidation in horizontal direction, we will use the time constant bound with the width or length of the consolidated area. Because horizontal flow of water may be taking place simultaneously lengthwise and widthwise of the consolidated area, considering this, and additionally assuming that horizontal distribution of stress under the stockpile is not changing, we may then define:
\[ s = \int_0^H \frac{\sigma - u}{M} \, dz = s_m \cdot \left[ 1 - F_1\left( \frac{t}{T_{0L}} \right) \cdot F_2\left( \frac{t}{T_{0B}} \right) \right] \]  

(6)

where:

\[ F_1\left( \frac{t}{T_{0L}} \right) = \frac{4}{\pi} \sum_{n=0}^\infty \left[ \frac{1}{(2n+1)^2} \right] \cdot \exp \left[ -(2n+1)^2 \cdot \frac{\pi^2}{4} \cdot \frac{t}{T_{0L}} \right] \cdot \sin \left( (2n+1) \frac{\pi \cdot y}{2L} \right) \]  

(7)

and

\[ F_2\left( \frac{t}{T_{0B}} \right) = \frac{4}{\pi} \sum_{n=0}^\infty \left[ \frac{1}{(2n+1)^2} \right] \cdot \exp \left[ -(2n+1)^2 \cdot \frac{\pi^2}{4} \cdot \frac{t}{T_{0B}} \right] \cdot \sin \left( (2n+1) \frac{\pi \cdot x}{2B} \right) \]  

(8)

Assuming time-related values of respectively the shear modulus and hydraulic conductivity:

\[ M(s) = M_0 \cdot \left( 1 - \frac{s}{n_0 \cdot H_0} \right)^{-\kappa} \]  

(9)

\[ k(s) = k_0 \cdot \left( 1 - \frac{s}{n_0 \cdot H_0} \right)^{-\kappa_f} \]  

(10)

In the equation (6) we may introduce a final subsidence with variable parameters of hydraulic conductivity and shear modulus:

\[ s_m(\sigma) = n_0 \cdot H_0 \cdot \left[ 1 - \left( 1 + \frac{\kappa - 1 \cdot \sigma}{n_0 \cdot M_0} \right)^{-\frac{1}{\kappa - 1}} \right] \]  

(11)

The runs of subsidence in the centre of a stockpile and at its perimeter, are important for final conclusions. For calculations the following soil parameters were assumed: \( L = B = 200 \text{ m} \), \( M_0 = 200 \text{ kPa} \), \( k_0 = 10^{-6} \text{ m/s} \) and in the second series \( k_0 = 10^{-5} \text{ m/s} \), thickness of the consolidated layer \( H = 8 \text{ m} \) and coefficients in equations (9) and (10): respectively \( \kappa = 1.86 \) and \( \kappa_f = 7.5 \). The assumed soil parameters correspond with conditions of overburdened peat soil in the Odra River Valley near the City of Szczecin, in the north-west of Poland.
In Fig. 3. the shape of the consolidated area is shown with system of coordinates used in calculations. In Fig. 4. is presented subsidence of overburdening layer of ash-slag mixture at different times using a simplified model with $M, k = \text{const}$.

**Fig. 3. Area subjected to forced consolidation**

**Fig. 4. Results of calculation of subsidence of overburdening embankment at constant soil parameters ($M, k = \text{const}$)**
Fig. 5. Results of calculation of subsidence of overburdening embankment at variable soil parameters: $M$ according to (9), $k$ according to (10).

In Fig. 5, the subsidence of the consolidated layer overburdened with ash-slag pile is illustrated, using calculations method incorporating changing soil parameters with time of consolidation, according to equations (9) and (10). In Fig. 6, the subsidence of the central point of the consolidated area $(x = 0, \; y = 0, \; z = H)$ is compared in time according to the model of vertical filtration (Terzaghi) and according to the discussed model of horizontal filtration.

Fig. 6. Comparison of subsidence in models of vertical and horizontal filtration.
3. Conclusion

1. A method of calculating the subsidence of a consolidated layer was presented, when vertical filtration of escaping groundwater is not possible due to the impermeability of the overburdening layer and horizontal filtration takes place.
2. Completed investigations indicate, that similar overburdening methods, allowing only for horizontal filtration, will result in weak soil layer being consolidated in much longer time than in case of vertical filtration.
3. Based on the completed analyses it is possible to state, that in case of horizontal filtration under the overburdening area firstly perimeter zones will subside, and finally the central part.
4. Results of calculations made according to a model with constant soil parameters compared with similar results with parameters changing in time, are indicating, that in the latter case the subsidence is much smaller, but consolidation takes longer. Confronting the results of calculations with observations made on cases in Szczecin, it is possible to indicate, that the second model is closer to the real phenomenon.
5. Further investigations should include solutions allowing for shortening the time of consolidation, for example by placing the overburdening pile of ash-slag mixture on a geo-textile layer speeding up evacuation of excess groundwater, or placement of drains.

4. Reference

5. **List of used symbols**

- $B$ – width of stockpile,
- $H_0$ – thickness of consolidated layer,
- $k$ – hydraulic gradient at Darcy’s equation,
- $L$ – length of stockpile base,
- $M_0$ – initial shear modulus,
- $M(s)$ – shear modulus during consolidation,
- $n_0$ – soil porosity,
- $s(t)$ – subsidence,
- $s_\infty$ – target subsidence,
- $t$ – time,
- $T_0$ – time constant,
- $X,Y,Z$ – system of coordinates,
- $\kappa$ – exponential root in equation of change of shear modulus,
- $\kappa_f$ – exponential root in equation of change of hydraulic conductivity,
- $\gamma_w$ – volume gravity of water,
- $\sigma$ – stress burdening the consolidated layer.